Simulation Analysis of Bidding Behavior in Sponsored Search Auction

Michiharu MASUI

Department of Business Administration, Ishinomaki Senshu University, Ishinomaki 986-8580

Abstract

Sponsored links (P4P ads) are a kind of Internet ads and they are sold at auctions, called sponsored search auctions. In this advertising system, the strategic behavior of bidders affects the result of auction. We examine how the biding pattern affect the result of sponsored search auction by agent-based simulation. Using our simulation, we find that bids converge though time and the revenue come close to the equilibrium revenue gradually where a robot cares about not only the bidder ranked one position above him but also bidders in lower positions. Implications of these findings are discussed in light of attention to way to construct the effective bidding on sponsored search auction and to design the new auction system.

1 Introduction

1.1 Sponsored Search Auction

Sponsored search ads are a kind of Internet ads and they are sold at auctions, called sponsored search auctions. Presently, major search engines employ the generalized second price auction (GSP), in which ad spots are allocated to the advertisers according to the descending order of the bids and each bidder pays the next lower bid per click when a search engine user clicks the bidder's link. Figure 1.1 shows the result of Yahoo and Figure 1.2 shows that of google. In Edelman et al.[(1)], they defined the locally envy-free equilibrium where each player cannot improve his payoff by exchanging bids with the bidder ranked one position above him. They showed that the revenue of search engines in any locally envy-free equilibrium in the GSP is higher than or equal to the revenue of the dominant strategy equilibrium in the Vickrey-Clarke-Groves (VCG) mechanism which has a incentive compatibility. In Fukuda, Masui and Ito[(3)], they conducted laboratory experiments to observe how bidders behave in the GSP and in the VCG. They reported many locally envy-free equilibria were achieved in the GSP, and the revenue of search engines in the GSP is higher than that in the VCG. In reality, each company uses a computer program, called a robot, that bids automatically according to historical datas of bids and its own revenue. In this paper, we consider some types of robot based on the results of the laboratory experiments and conduct computer simu-lation ex-

Figure 1.1: Yahoo

Figure 1.2: Google

periments. In these simulation experiments, we can construct a long term market of sponsored search ads while in the laboratory experiments each session has only 10 auction periods. We observe that bids converge though time and the revenue come close to the equilibrium revenue gradually where a robot cares about not only the bidder ranked one position above him but also bidders in lower positions. And we analyze behavioral criteria of bidders or the reference points of bidders. Moreover, we discuss which auction mechanism is preferable for search engines in real market.

1.2 Previous Works

Theoretical work

In Edelman et al.(1), they defined the locally envy-free equilibrium where each player cannot improve his payoff by exchanging bids with the bidder ranked one position above him. They showed that the revenue of search engines in any locally envy-free equilibrium in the GSP is higher than or equal to revenue of the dominant strategy equilibrium in the Vickrey-Clarke-Groves (VCG) mechanism.

Experimental work

In Fukuda et al.(3), they conducted laboratory ex-periments to observe how bidders behave in the GSP and in the VCG. They reported many locally envy-free equilibria were acheived in the GSP, and the revenue of search engines in the GSP is higher than that in the VCG.

1.3 Motivation and Purposes

In Fukuda et al.(3), they observed that there are many bidding patterns and many cases where the results are different from theoretical. Therefore we consider some types of bidders based on the laboratory experiment and make the computer simulation to study the bidding pattern of these types.

2 Model

2.1 Notations

An auction game has the following components. Let $N=\{1, 2, \ldots, n\}$ be the set of bidders in auction. Each bidder i has the evaluation of ad effectiveness or expected revenue x_i for a click of the ad. Note that the evaluation are spot-independent. There are K ad-spots with click-through rates (CTRs). CTRs is the estimated number of clicks per a constant time. We assume that CTRs are bidder-independent and $\alpha_1 > \alpha_2 > \ldots > \alpha_K > 0$. In reality, however, CTR of the bottom spot α_K is almost 0. We denote the bid profile of N bidders by $b=(b_1, b_2, \ldots, b_n)$. An auction mechanism determines how ad-spots are allocated to bidders and the payment of bidders for a click.

2.2 Generalized Second Price Auction

In this paper, we mainly refer to generalized second price auction (GSP). GSP is represented by following rules.

- Slots are allocated in the decending order of the bids. Let $d(k)$ denote the name of the bidder who obtains the k-th spot. Note that in our setting, $d(K+1)$ is not defined, and thus, we set $b_{d(K+1)}=0$
- If $i=d(k)$, i's payment per click $p_k(b)$ is the next lower bid: $b_{d(k+1)}$. His/her total payment is $p_i(b)=\alpha_k b_{d(k+1)}$.
- If $i=d(k)$, his/her utility is $u_i(b)=\alpha_k x_i - \alpha_k b_{d(k+1)} = \alpha_k(x_i - b_{d(k+1)})$

Locally envy-free equilibrium

In general, there are multiple Nash equilibria in the GSP. In Edelman et al.(3), they defined a restricted notion of equilibrium called locally envy-free. b is locally envy-free if no bidder can improve his/her utility by exchange bids with the bidder ranked one position above him/her. Formally, for any $k \le K = n$,

$\alpha_k x_i - p_i(b) \geq \alpha_{k-1} x_i - p_{d(k-1)}(b)$,
where $i = d(k)$

In Fukuda et al., they observed many locally envy-free equilibria are acheived. However, they also observed that bidders changed his/her bid even if some locally envy-free equilibria were acheived.

3 Simulation Design

This section describes the simulation design. There are various types of bidding pattern in the real market. In this paper, however, we construct the following types of bidder.

Upward (U) If $\alpha_k x_i p_i(b) < \alpha_{k-1} x_i - p_{d(k-1)}(b)$ then i change bid to $b_{d(k-1)}$. Otherwise i bids b_i.

Downward (D) If $\alpha_k x_i p_i(b) < \alpha_{k+1} x_i - p_{d(k+1)}(b)$ then i change bid to $b_{d(k+1)} - 1$. Otherwise i bids b_i.

Up and Down (UD) Bidder i calculates $\alpha_{k-1} x_i - p_{d(k-1)}(b)$ and $\alpha_{k+1} x_i - p_{d(k+1)}(b)$. If $\alpha_k x_i p_i(b) < \alpha_{k+1} x_i - p_{d(k+1)}(b) < \alpha_{k-1} x_i - p_{d(k-1)}(b)$ then i change bid to $b_{d(k-1)}$. If $\alpha_k x_i p_i(b) < \alpha_{k-1} x_i - p_{d(k-1)}(b) < \alpha_{k+1} x_i - p_{d(k-1)}(b)$ then i change bid to $b_{d(k+1)} - 1$. Otherwise i bids b_i.

Highest (H) Each bidder calculate the payoff of each spot. Then i change bid to the most profitable spot: If the most profitable spot for i is l then i changes bid to $b_{d(i-1)}$.

Due to the space limitation, we introduce the result of the several patterns in this paper. There are 5 bidders and 5 ad-spots. The case in which each bidder is the same type is expressed as "ALL-(type name) case". On the other hand, the case which there are several types of bidders is "Mix case"; we consider, for example, the case in which there are 1 U-type, 2 D-types, 1 UD-type, and 1 H-type. We de-note this case "Mix-(1,2,1,1) case". The bidding range is also important parameter. We conjecture that the bidding price strongly affect the result of auction. Therefore, we determine the x_i from five range: "Low(1-100)", "Middle(200-400)", "High (600-800)", "Lower-Wide(1-500)", and "Upper-Wide(500-1000)". Initial bid is selected randomly from $[x_i - \alpha, x_i + \alpha]$, where α is 0, 10, 25, or 50. The CTRs are commonly known to the bidders and fixed at (100,80,50,40,20). In addition, we repeats the auction 500 times. Parameters are summarized in Table 3.1.

Table 3.1: Parameters of simulation

Num. of bidders	5
Num. of ad-spots	5
Repeat	500
Values	Low, Middle, High, L-Wide, U-Wide
CTRs	(100, 80, 50, 40, 20)

Our simulation runs in the following steps.

Step 1 Set the price range, the type of bidder, value, and initial bid.

Step 2 Assign the ad-spot and calculate the pay-off and payment based on the previous state.

Step 3 Each bidder recalculates the bidding price.

Step 4 Repeat Step 2 and Step 3 for 500 times. After that, stop the simulation.

4 Results

4.1 "ALL" case

We refer to some remarkable observations in "ALL" cases.

In ALL-U case, once the bidding of all bidders converge, all bidders remain the current state. Figures 4.1 and 4.2 show the bidding pattern in ALL-D and ALL-UD. The bidding price gradually decreases and, after sufficient time, converge on the most profitable state of the bidder. Tables 4.1 and 4.2 show the average payoff and the average revenue of 50 times of simulation. The bidders in ALL-H bid

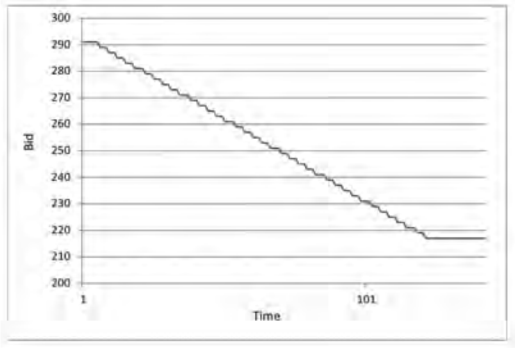

Figure 4.1: Bidding pattern in ALL-D (Middle)

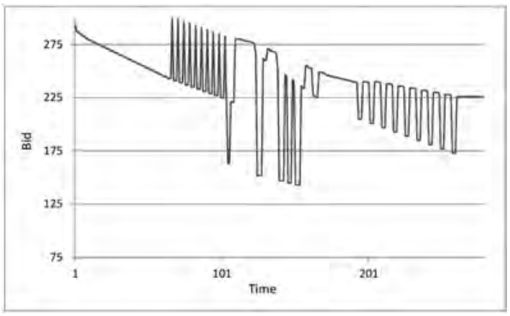

Figure 4.2: Bidding pattern in ALL-UD (Middle)

efficiently from the viewpoint of the average payoff. Conversely, the revenue of search engine in ALL-H is lower than other cases.

Table 4.1: Average payoff

	ALL-U	ALL-D	ALL-UD	ALL-H
Low	1194	2349	2644	1459
Middle	2360	8607	6900	7663
High	4722	14588	10737	21699
L-Wide	3122	11485	8771	9245
U-Wide	9158	18476	14774	20731

Table 4.2: Average revenue

	ALL-U	ALL-D	ALL-UD	ALL-H
Low	19940	15840	6175	8708
Middle	66950	46984	62134	40159
High	191290	147945	153577	111942
L-Wide	51280	33585	34253	39835
U-Wide	182320	138520	175878	126104

We observe the bidding pattern of our simulation expresses the bidding pattern in the real market. Figure 4.3 shows the result of ALL-H case in the middle price range. It is similar to the graph in Figure 4.4 that describes the "Sawtooth pattern"(2). In addition, Figure 4.2 is also similar to the Sawtooth pattern.

Table 4.3: Average revenue

(U,D,UD,H)	(1,1,1,2)	(1,1,2,1)	(1,2,1,1)	(2,1,1,1)
Low	7194	9808	7639	9840
Middle	58278	59153	56850	69339
High	135271	149473	147622	160043
L-Wide	32009	42635	36180	60545
U-Wide	159595	169130	154823	178920

4.2 "Mix" case

In the Mix cases, the U-type bidders have an effect on the bidding and the revenue. Table 4.3 shows the average revenue of 50 simulations. The average revenue of Mix-(2,1,1,1) cases is greater than other cases. It is thought that the U-type pulls up the bidding price of the UD-type and the H-type; whereas the D-type bidder pulls down the bid price, however, the effect of lifting is stronger than the effect of bringing down. It is shown in Figure 4.5. We observe this tendency in other cases. In contrast, in the case of one U-type bidder, there are no excessive rise and fall of a bid.

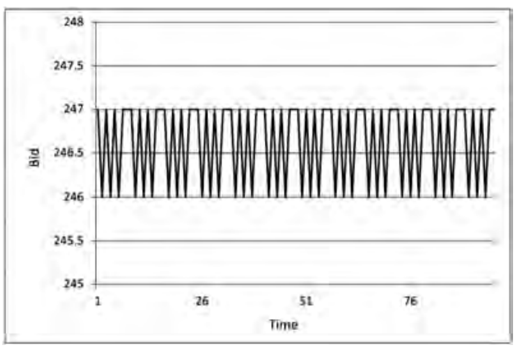

Figure 4.3: Bidding pattern in ALL-H (Middle)

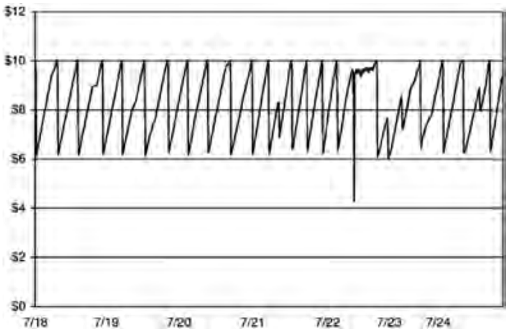

Figure 4.4: "Sawtooth" pattern(2)

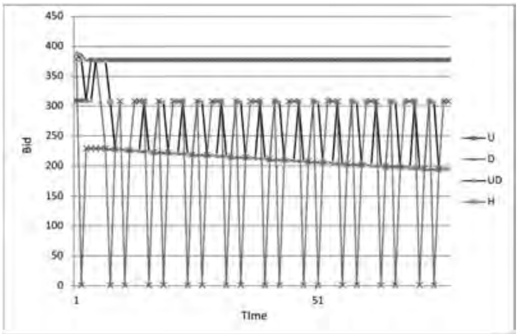

Figure 4.5: Bid profile in Mix case (Middle)

5 Conclusion

In this study, we construct the simulation and examine the bidding behavior in sponsored search auction. We observe that the bidding pattern of the UD-type or the H-type is similar to that in the real market. This observation indicates that bidders in the real market determine the bidding price based on the upper and lower bidding price rather than own criterion. In addition, we observe that the revenue is affected by the bidding pattern. An ideal mechanism has the robustness to the bidding pattern. Since the VCG has a incentive compatibility, it is well-known mechanism as the robust mechanism. However, the VCG is not put in practical use. This is why the payment rule of the VCG is too complex for bidders. We will consider the simple and robust mechanism and, to examine the performance of the mechanism, we will construct a simulation considering other types of decision maker.

References

(1) Edelman, B. and M.Ostrovsky (2006), “Strategic Bidder Behaviour in Sponsoired Search Auction”, *Decision Support System*, **43**(1),192-198.

(2) Edelman, B., M.Ostrovsky and M. Schwarz (2007), “Internet advertising and the Generalized Second Price Auction: Selling Billions of Dollars Worth of Keywords,” *American Economic Review*, **97**(1),242-259.

(3) Fukuda, E., M. Masui and N. Ito (2007), “An Experimental Study of Sponsored Search Auctions” (in Japanese), *IEICE Technical Report*, **107**, AI2007-24, 23-26.